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GRAPHICAL SOLUTION OF CUBIC EQUATIONS.

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Every numerical cubic equation, whose coefficient are real, which is not of the form $x^3 - a x + b = 0$ or $x^3 + a x + b = 0$, where the intrinsic value of a is positive, but b either positive or negative, may be reduced to one of these forms by known rules.

If a is not zero, the first of the above equations may be transformed into

$$x'^3 - x' + \frac{b\sqrt{a}}{a^2} = 0$$

by substituting for x , $\sqrt{a} \cdot x'$, dividing the resulting equation by $a\sqrt{a}$, and afterwards making the denominator of the constant term rational. We put y equal to the left hand member of this last equation and plot the curve when the term $b\sqrt{a} \div a^2$ is zero.

Now, whatever the value of $b\sqrt{a} \div a^2$, the same curve will satisfy the equation provided the origin is removed upon the axis of y a distance equal to the numerical value of this quantity with its sign changed. We draw a line through this new position of the origin parallel to the primitive axis of x' . The real roots of the equation

$$x'^3 - x' + \frac{b\sqrt{a}}{a^2} = 0$$

are the distances on this line intercepted between its intersection with the curve and the new position of the origin, and may be determined by careful measurement with a good degree of accuracy upon a well drawn chart. The values of x may be computed from the equation $x = \sqrt{a} \cdot x'$. The equation $x^3 + a x + b = 0$ may be transformed into

$$x'^3 + x' + \frac{b\sqrt{a}}{a^2} = 0$$

and its real root found for any value of $b \sqrt{a} \div a^2$ by a process similar to that employed in

$$x'^3 - x' + \frac{b \sqrt{a}}{a^2} = 0.$$

Both the real and imaginary roots of cubic equations may be found with considerable accuracy by means of a table constructed from the equations

$$x'^3 - x' + \frac{b \sqrt{a}}{a^2} = 0 \text{ and } x'^3 + x' + \frac{b \sqrt{a}}{a^2} = 0$$

in the following manner : In

$$x'^3 - x' + \frac{b \sqrt{a}}{a^2} = 0$$

we can put for $b \sqrt{a} \div a^2$, $\pm c (c^2 - 1)$ where $\mp c$ is a root of the equation. Dividing $x'^3 - x' \pm c (c^2 - 1)$ by $x' \pm c$ we obtain $x'^2 \mp c x' + (c^2 - 1)$, which put equal to zero and solved as a quadratic gives the other roots. Beginning with some small quantity and assuming in succession values for c differing from each other as little as we please, a table can be computed showing the values of $b \sqrt{a} \div a^2$ and the three corresponding roots to any degree of accuracy.

When $b \sqrt{a} \div a^2 = \pm .385 +$ we have the case of two equal roots, and for greater values of $b \sqrt{a} \div a^2$ there is but one real root, and the other two are imaginary.

Designating the pair of imaginaries corresponding to the real root $+c$ by $-A \pm B \sqrt{-1}$, we may compute the values of A and B for the table. The imaginary roots answering to the various values of $b \sqrt{a} \div a^2$ may be represented on the chart. By the principles of the General Theory of Equations the sum of the three roots of the equation

$$x'^3 - x' + \frac{b \sqrt{a}}{a^2} = 0$$

is equal to zero ; hence the A of the imaginary is equal to one-half the real root with the opposite sign, and may be laid off on the same line as the real root but on the opposite side of the axis of y . If, at the extremity of this line, a perpendicular be drawn and B be laid off upon this perpendicular in both directions, we have a complete representation of the imaginary roots. By laying off a sufficient number of these values, we are able to draw the curve which represents the imaginary roots.

It will be observed that as the two roots on the left of the axis of y

approach equality their values change rapidly in comparison with the value of $b\sqrt{a} \div a^2$. The same holds true for the value of B in the case of the first imaginaries. But as $b\sqrt{a} \div a^2$ increases, the changes in the real and imaginary roots grow relatively smaller, and results of great accuracy may be obtained by interpolation.

In the equation

$$x'^3 + x' + \frac{b\sqrt{a}}{a^2} = 0$$

the computation of $b\sqrt{a} \div a^2$ for the table may be abridged, by taking into account the fact that the third order of differences in the series of terms $b\sqrt{a} \div a^2$ corresponding to the values of $c = .1, .2, .3$ &c. vanishes, and B of the imaginaries varies very uniformly.

To use the table for numerical computation, reduce the equation to one of the two forms

$$x'^3 - x' + \frac{b\sqrt{a}}{a^2} = 0 \text{ or } x'^3 + x' + \frac{b\sqrt{a}}{a^2} = 0,$$

compute $b\sqrt{a} \div a^2$ by means of a table of squares and square roots, find this number in the proper table in the column

$$\frac{b\sqrt{a}}{a^2},$$

and opposite will be found the roots, if real. If there is a pair of imaginaries the columns A and B will furnish their values. If $b\sqrt{a} \div a^2$ is not found in the table, find the next less number, and add to the corresponding roots proportional parts.

The following is a portion of a table constructed as described above :

TABLE.

$$x'^3 - x' + \frac{b\sqrt{a}}{a^2} = 0$$

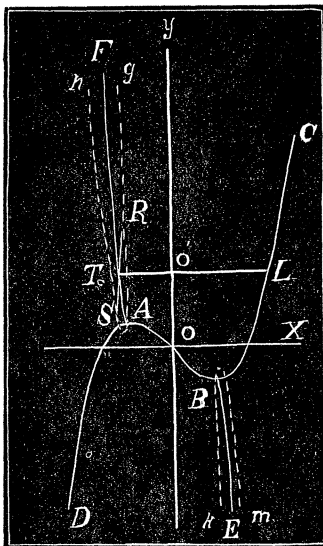
$\pm \frac{b\sqrt{a}}{a^2}$	REAL ROOTS.			$\pm \frac{b\sqrt{a}}{a^2}$	$A.$	$B.$	REAL ROOTS.
$\pm .000$	$\pm .000$	± 1.000	∓ 1.000	$\pm .386+$	$\pm .5775$	$.024+$	∓ 1.155
.099	.100	.946+	1.046+	.401+	.580	.096+	1.160
.192	.200	.885+	1.085+	.463+	.590	.215+	1.180
.273	.300	.816+	1.116+	.528+	.600	.283+	1.200
.336	.400	.738+	1.138+	.595	.620	.341+	1.220
.359	.450	.696+	1.146+	.817	.640	.479+	1.280
.375	.500	.651+	1.151+	.980	.660	.554+	1.320
.377+	.510	.642+	1.152+	1.155	.680	.622+	1.360
.379+	.520	.633+	1.153+	1.344	.700	.685+	1.400
.381+	.530	.624+	1.153+	2.496	.800	.959+	1.600
.383+	.540	.614+	1.154+	4.032	.900	1.196+	1.800
.384+	.550	.602+	1.154+	6.000	1.000	1.414+	2.000
.385+	.577+	.577+	1.154+	24.000	1.500	2.398+	3.000

$$x'^3 + x' + \frac{b\sqrt{a}}{a^2} = 0.$$

$\pm \frac{b\sqrt{a}}{a^2}$	A.	B.	REAL ROOT.	$\pm \frac{b\sqrt{a}}{a^2}$	A.	B.	REAL ROOT.
.000	±.000	±1.000	∓.000	2.000	±.500	±1.323 +	∓1.000
.101	.050	1.004 +	.100	2.431	.550	1.381 +	1.100
.208	.100	1.015 +	.200	2.928	.600	1.442 +	1.200
.327	.150	1.033 +	.300	3.497	.650	1.506 +	1.300
.464	.200	1.058 +	.400	4.144	.700	1.572 +	1.400
.625	.250	1.090 +	.500	4.875	.750	1.640 +	1.500
.816	.300	1.128 +	.600	5.696	.800	1.709 +	1.600
1.043	.350	1.170 +	.700	6.613	.850	1.780 +	1.700
1.312	.400	1.216 +	.800	7.632	.900	1.852 +	1.800
1.629	.450	1.268 +	.900	10.000	1.000	2.000	2.000

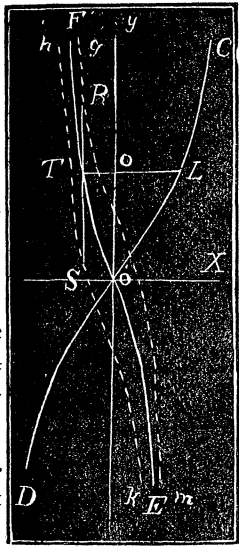
$$x'^3 - x' + \frac{b\sqrt{a}}{a^2} = 0$$

$$x'^3 + x' + \frac{b\sqrt{a}}{a^2} = 0$$



$C B A D =$ curve whose equation is $y = x'^3 - x'$. $O' L =$ any real root. $O' T =$ the “A” of an imaginary. $T R$ and $T S =$ the “ $\pm B$ ” of an imaginary.

$C' O D' =$ curve whose equation is $y = x'^3 + x'$. $O' L =$ any real root. $O' T =$ the “A” of an imaginary. $T R$ and $T S =$ the “ $\pm B$ ” of an imaginary.



EXAMPLE. $x^3 - 3x - 3 = 0$.

Put $x = \sqrt[3]{3} \cdot x'$ and substitute, $3\sqrt[3]{3} \cdot x'^3 - 3\sqrt[3]{3} \cdot x' - 3 = 0$.

Dividing by $3\sqrt[3]{3}$ and reducing, gives $x'^3 - x' - .577 + = 0$, where $-.577 + = \frac{b\sqrt{a}}{a^2}$. The real root in the table answering to the next less value of $\frac{b\sqrt{a}}{a^2}$.528 is 1.20. By proportional parts $x' = 1.214 +$. Proceeding in a similar way the imaginary roots may be found, and by substituting the values of x' in the equation $x = \sqrt[3]{3} \cdot x'$, the values of x may be obtained.